## COMMON CORE STANDARDS ADDRESSED IN THIS RESOURCE

N-CN. 4 - Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
Activity pages: 36, 37, 38
F-IF. 7 - Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
Activity pages: 23, 24
F-TF. 1 - Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. Activity pages: 3, 4

F-TF. 2 - Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
Activity pages: 7, 10
F-TF. 5 - Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
Activity pages: 19, 20, 21, 22
F-TF. 7 - Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
Activity pages: 14, 17, 18
F-TF. 8 - Prove the Pythagorean identity $\sin 2(\theta)+\cos 2(\theta)=1$ and use it to find $\sin (\theta), \cos (\theta)$, or $\tan (\theta)$ given $\sin (\theta)$, $\cos (\theta)$, or $\tan (\theta)$ and the quadrant of the angle.
Activity pages: 17, 18
F-TF. 9 - Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. Activity pages: 15, 16, 26

G-SRT. 6 - Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
Activity pages: 5, 9, 11, 12
G-SRT. 8 - Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
Activity pages: 6, 8, 13
G-SRT. 9 - Derive the formula $A=1 / 2 a b \sin (C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
Activity page: 32
G-SRT. 11 - Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
Activity pages: 30, 31, 33, 34

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$\qquad$

## Remember

1. An angle is formed by two rays with a common endpoint.

The size of an angle is the amount of rotation between its two rays.
2. Counterclockwise rotation is positive. Clockwise rotation is negative.
3. Units for measuring rotation are
revolution, degree, radian, and grad (gradian or gradient).

4. Conversion Formulas:

1 revolution $=360^{\circ}$
$\pi$ radians $=180^{\circ}$
100 grads $=90^{\circ}$

## Example:

Example:
Convert $60^{\circ}$ to radians. $\quad 60^{\circ} \times \frac{\pi \text { radians }}{180^{\circ}}=\frac{\pi}{3}$ radians

Play "odd measure out." Cross through the measure in each row that is not equivalent to the other three.

1.

| 1 revolution | $360^{\circ}$ | $\pi$ radians | 400 grads |
| :---: | :---: | :---: | :---: |

2. | $\frac{\pi}{3}$ radians | $\frac{1}{12}$ revolution | $66 \frac{2}{3}$ grads | $60^{\circ}$ |
| :---: | :---: | :---: | :---: |
3. | $133 \frac{1}{3}$ grads | $\frac{2 \pi}{3}$ radians | $\frac{2}{3}$ revolution | $120^{\circ}$ |
| :---: | :---: | :---: | :---: |
4. 

| $210^{\circ}$ | $233 \frac{1}{3}$ grads | $\frac{7 \pi}{3}$ radians | $\frac{7}{12}$ revolution |
| :---: | :---: | :---: | :---: |

5. 


6. 1,200 grads

$\qquad$ 3 revolutions 6 radians
7. $\frac{3}{2 \pi}$ revolution 3 radians
$\left(\frac{540}{\pi}\right)^{\circ}$ $\frac{\pi}{600}$ grad

## Remember

The linear measure, $s$, of an arc of a circle is related to the radian measure of its central angle, $\theta$, and the radius, $r$.

$$
s=\theta r
$$



Example: Find the length of the arc that subtends an angle of $40^{\circ}$ in a circle whose radius is 18 inches. Answer to the nearest tenth of an inch.

1. Convert the angle measure from degrees to radians.

$$
40^{\circ} \times \frac{\pi \text { radians }}{180^{\circ}}=\frac{2 \pi}{9} \text { radians }
$$

2. Substitute the radian value for $\theta$

$$
s=\theta r
$$

and the value of the radius
into the arc-length formula.
$s=\frac{2 \pi}{9} \times 18$
3. Substitute for $\pi$, and evaluate.

$$
s \approx \frac{2(3.14)}{9} \times 18 \approx 12.56 \approx 12.6 \text { inches }
$$

Find the indicated measures.
Then use the answer code to complete the seven-word sentence below.

1. In a circle with radius 12 centimeters, find the length of an arc intercepted by a central angle of $45^{\circ}$.
2. A circle has a radius of 6 feet. Find the radian measure of a central angle that intercepts an arc length of 12 feet.
3. In a circle, a central angle of $30^{\circ}$ intercepts an arc of 23.5 inches. Find the length of the radius.
4. The length of a pendulum is 18 inches. Find the distance through which the tip of the pendulum travels when the pendulum turns through an arc of 2.5 radians.
5. The diameter of a wheel is 48 inches. Find the number of degrees through which a point on the circumference turns when the wheel moves a distance of 2 feet.

| $28.7^{\circ}$ | theta |
| :--- | :--- |
| $57.3^{\circ}$ | pi |
| 44.9 in. | could |
| 45 in. | calculate |
| $2 \pi$ | would |
| 2 | wish |
| 0.5 | recalculate |
| $3 \pi \mathrm{~cm}$ | how |
| 3 cm | think |



Count the letters in each of the seven words. Use the count to fill in the spaces below, revealing the values of the first seven digits of $\pi$.

Name $\qquad$ Trigonometric Functions of Acute Angles

## Remember

1. There are 6 trigonometric functions: sine (sin), cosine (cos), tangent (tan), cosecant (csc), secant (sec), and cotangent (cot)
2. Cofunctions: sine and cosine, tangent and cotangent, secant and cosecant
3. Reciprocal Functions: (sine, cosecant), (cosine, secant), (tangent, cotangent)
4. In a right triangle, the hypotenuse is always opposite the right angle.

In right triangle $A B C$, legs $a$ and $b$ are named with respect to acute angle $A$.
$\sin A=\frac{a}{c}, \csc A=\frac{c}{a}$
$\cos A=\frac{b}{c}, \sec A=\frac{c}{b}$
$\tan A=\frac{a}{b}, \cot A=\frac{b}{a}$


## Across

3. $\sin A=$ $\qquad$ B
4. relationship between the acute angles of a right triangle
5. $\sin A=\frac{1}{\square} A$
6. $\qquad$ $A=\cot B$
7. opposite
8. $\frac{\text { hypotenuse }}{\text { adjacent }}$
9. $\frac{\text { opposite }}{\text { hypotenuse }}$

## Down

1. $\frac{\text { hypotenuse }}{\text { opposite }}$
2. side opposite right angle
3. $\quad \_A=\frac{1}{\tan A}$
4. $\quad A=\frac{1}{\cos A}$
5. $\frac{\text { adjacent }}{\text { hypotenuse }}$
6. product of two reciprocals
7. $\frac{\text { adjacent }}{\text { opposite }}$
