

# COMMON CORE STANDARDS ADDRESSED IN THIS RESOURCE

N-CN.9 - Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Activity pages: 34, 35

A-APR.2 - Know and apply the Remainder Theorem: For a polynomial  $p(x)$  and a number  $a$ , the remainder on division by  $x - a$  is  $p(a)$ , so  $p(a) = 0$  if and only if  $(x - a)$  is a factor of  $p(x)$ .

Activity page: 30

A-APR.3 - Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Activity page: 31

A-APR.6 - Rewrite simple rational expressions in different forms; write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.

Activity pages: 28, 29

A-REI.4 - Solve quadratic equations in one variable.

Activity pages: 30, 31, 32, 33

F-IF.1 - Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

Activity page: 3

F-IF.2 - Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Activity pages: 5, 20, 21

F-IF.3 - Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

Activity pages: 4, 36, 37, 38, 39, 40, 41

F-IF.7 - Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

Activity pages: 11, 12, 17

F-BF.1 - Write a function that describes a relationship between two quantities.

Activity pages: 32, 33

F-BF.3 - Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Activity pages: 6, 7, 8, 9, 10, 13, 14

F-BF.4 - Find inverse functions.

Activity pages: 15, 16

F-BF.5 - Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Activity pages: 22, 23

F-LE.4 - For exponential models, express as a logarithm the solution to  $ab^ct = d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.

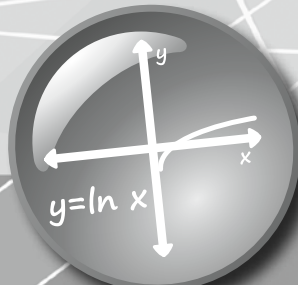
Activity pages: 18, 19

F-LE.5 - Interpret the parameters in a linear or exponential function in terms of a context.

Activity pages: 24, 25, 26, 27

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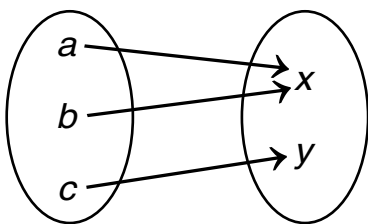


**Remember**

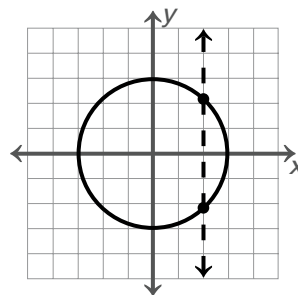
1. A **relation** is a set of *ordered pairs*. The **domain** is the set of all first elements of the ordered pairs and the **range** is the set of all second elements.
2. A **function** is a relation in which each member of the domain corresponds to exactly one member of the range.
3. To determine if a relation specified by a *mapping diagram* is a function, use the arrows to find the pairings. To be a function, each domain-value should have only one arrow starting from it. It does not matter if a range-value has more than one arrow coming to it.
4. To determine if a relation specified by a *graph* is a function, apply the **vertical-line test**. For a function, any vertical line drawn through the graph must intersect the graph in exactly one point.
5. When a relation is specified as a *rule* that states the correspondence between two variables, the domain is the set of allowable real values for the *input-value* and the range is the set of resulting real values for the *output-value*.

**Examples**

1. Determine if each relation is a function.



Each domain-value is paired with exactly one range-value:  $(a, x)$ ,  $(b, x)$ ,  $(c, y)$ . This mapping diagram does not represent a function.



Except at the  $x$ -intercepts, any vertical line will intersect this circle in more than one point. This graph does not represent a function.

2. Determine the domain and range of each function.

$$y = \sqrt{x-1}$$

This radicand must be  $\geq 0$ .  
 Domain =  $\{x \mid x \geq 1\}$   
 [read: *the set of elements  $x$  such that  $x$  is greater than or equal to 1*]

$y$  is a principal square root. Range =  $\{y \mid y \geq 0\}$

$$y = \frac{4}{x-2}$$

The denominator cannot be 0.  
 Domain =  $\{x \mid x \neq 2\}$   
 Express  $x$  in terms of  $y$ .  
 (cross multiply; solve for  $x$ )

The denominator cannot be 0.  
 Range =  $\{y \mid y \neq 0\}$

$$y = \frac{4}{x-2}$$

$$xy - 2y = 4$$

$$x = \frac{4 + 2y}{y}$$

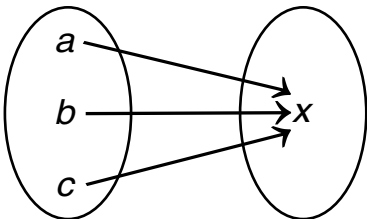
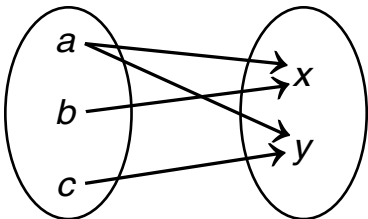
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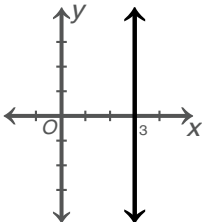
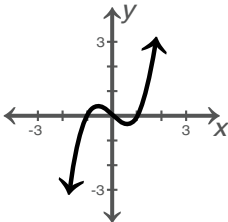
# Identifying Functions

Choose the best answer and write its letter in the answer column.

1. The relation  $\{(0, 7), (-2, 7), (1, 5), (5, 1), (0, -1), (-1, 0)\}$  is not a function. Which of the ordered pairs listed below, if omitted from this relation, will make the resulting set a function?  
**(a)** (1, 5)    **(b)** (5, 1)    **(c)** (0, -1)    **(d)** (-1, 0)    **(e)** (-2, 7)    1. \_\_\_\_\_

2. A function is defined by the equation  $y = -3x^2 + 2$  and its domain is limited to  $\{x \mid -3 \leq x \leq -1\}$ . What is the greatest value in the range?  
**(a)** -25    **(b)** -1    **(c)** 0    **(d)** 1    **(e)** 25    2. \_\_\_\_\_

3. Which mapping diagram represents a relation that is *not* a function?  
**(a)**     **(b)**     3. \_\_\_\_\_

4. Which graph represents a function?  
**(a)**     **(b)**     4. \_\_\_\_\_

5. What is the domain of the function  $y = \frac{5}{\sqrt{x+1}}$ ?  
**(a)**  $\{x \mid x \neq -1\}$     **(b)**  $\{x \mid x \geq -1\}$     **(c)**  $\{x \mid x < -1\}$     **(d)**  $\{x \mid x > -1\}$     5. \_\_\_\_\_

6. What is the range of the function  $y = \frac{x}{x+3}$ ?  
**(a)**  $\{y \mid y \neq 0\}$     **(b)**  $\{y \mid y \neq 3\}$     **(c)**  $\{y \mid y \neq 1\}$     **(d)** {real numbers}    6. \_\_\_\_\_

Shade the cells of the grid that contain your six answers.

Together with those cells that are already shaded, you will see an operational symbol commonly used in modern mathematics that was first seen in the year 1489 in an arithmetic book written by a German mathematician, Johannes Widmann.

5(a)	2(a)	3(b)	5(c)	1(b)
1(a)	6(b)		6(a)	2(e)
	2(b)	5(d)	4(b)	6(c)
3(a)	5(b)	1(c)	2(c)	4(a)
2(d)	1(e)		6(d)	1(d)

**Remember**

1. A **function rule**, such as  $y = 3x$ , is an equation that describes a relationship between two variables.
2. **Function notation**, such as  $f(x) = 3x$ , is used to express a function rule. The symbol  **$f(x)$** , [read:  $f$  of  $x$ ], replaces  $y$  and shows that  $x$  is the *input-value* or **independent variable**.
3. To find the corresponding output-value for a specific input-value, evaluate the function rule at the indicated input-value. If  $f(x) = 3x$ , then  $f(5) = 3(5) = 15$ .
4. Different letters are used in function notation to distinguish between different functions of the same variable. For example:  $f(x) = 3x$  and  $g(x) = x^2$
5. For functions  $f$  and  $g$ , the **composite function  $f \circ g$**  is defined as  **$(f \circ g)(x) = f(g(x))$** , [read:  $f$  of  $g$  of  $x$ ]. This composite has  $x$  in the domain of  $g$  and  $g(x)$  in the domain of  $f$ . Work with the inner function first when evaluating a composite (in this case, the  $g$ -function).

**Example** If  $f(x) = 3x$  and  $g(x) = x^2$ , then:

a. find  $f(g(4))$

Work with  $g$  first.

$$g(x) = x^2$$

Evaluate  $g$  at  $x = 4$ .  $g(4) = 4^2 = 16$

Evaluate  $f$  at  $x = 16$ .  $f(x) = 3x$

$$f(16) = 3(16)$$

$$f(g(4)) = 48$$

b. find  $g(f(4))$

Work with  $f$  first.

$$f(x) = 3x$$

Evaluate  $f$  at  $x = 4$ .  $f(4) = 3(4) = 12$

Evaluate  $g$  at  $x = 12$ .  $g(x) = x^2$

$$g(12) = 12^2$$

$$g(f(4)) = 144$$

**Interpret function notation to answer each question.**

1. If  $f(x) = |x-1|$  and  $g(x) = -x$ , then  $f(-5) + g(16) = \underline{\hspace{2cm}}$ .
2. If  $f(x) = \sqrt[3]{x}$  and  $g(x) = x^{-1}$ , then  $f(-8) \div g(2) = \underline{\hspace{2cm}}$ .
3. If  $f(x) = cx^2$  and  $f(2) = -24$ , then  $c = \underline{\hspace{2cm}}$ .
4. If  $f(x) = x^2 + x$  and  $g(x) = 6x + 6$ , for what positive value of  $x$  does  $f(x) = g(x)$ ?  $\underline{\hspace{2cm}}$
5. If  $f(x) = \frac{x}{2}$  and  $g(x) = x^2$ , for what value of  $x$  does  $f(g(x)) = g(f(x))$ ?  $\underline{\hspace{2cm}}$

**Evaluate each composite function.**

6.  $f(x) = -\sqrt{x}$

$$g(x) = x^3$$

$$f(g(4)) = \underline{\hspace{2cm}}$$

7.  $f(x) = \sqrt{x+1}$

$$g(x) = \sqrt{x}$$

$$f(g(255)) = \underline{\hspace{2cm}}$$

8.  $h(x) = x^{-\frac{1}{3}}$

$$j(x) = x^3$$

$$h(j(\frac{1}{2})) = \underline{\hspace{2cm}}$$

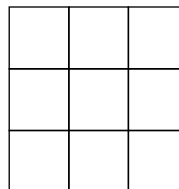
9.  $j(x) = |-x|$

$$k(x) = \sqrt[3]{-x}$$

$$k(j(-8)) = \underline{\hspace{2cm}}$$

A *magic square* has the same sum for every row, column, and diagonal.

Using your answers for all nine questions, create a magic square.



magic sum

=  $\underline{\hspace{2cm}}$